Retake Test 1 Num. Math. 2, May 11, 2023

In front of the questions one finds the weights used to determine the final mark.

Problem 1

a Consider the matrix

$$A = \begin{bmatrix} 1 & 1 \\ \epsilon & 0 \\ 0 & \epsilon \end{bmatrix}$$

for the problem $Ax = b$, with $\epsilon = 10^{-10}$. For this problem we want to find the least-squares solution.

a [6,8] [4] Show that it is not possible to solve this problem via the normal equations on a standard PC. having unit round-off error $\sim 10^{-16}$. Use rounding in the intermediate results

b [2] Create a QR factorization of A using the Gram-Schmidt proc. How is this QR factorization used to find the least squares solution and does this lead to a solvable system?

c [0.8] Compute the singular values of A.

Problem 2

2. Consider $Ax = b$ with A and b given by

$$A = \begin{bmatrix} 1 & 10^{-20} \\ 2 & 2\,10^{20} \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2\,10^{20} \end{bmatrix}.$$

Hence $x_2 = 1$ and $x_1 = 1 - 10^{-20} \approx 1$.

Let the unit roundoff be given by $u = 10^{-16}$. Below you have to use this to round the intermediate results. What will be the solution if we solve the linear system, including rounding, using, Gaussian Elimination

(a) [0.5] without pivoting,

(b) [0.75] with partial pivoting,

(c) [0.75] with complete pivoting,

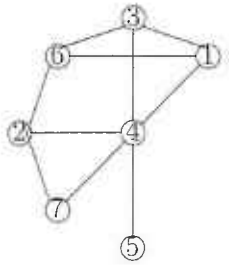
(d) [0.75] with partial pivoting, where a row scaling is applied such that the maximum on each row of the matrix is 1.

(e) [0.25] Which two approaches will, in general, give the correct result?

Questions continue on other side

Problem 3

3. Consider the graph of a symmetric matrix A depicted below.



(a) [0.4] Make a sketch of the associated matrix (with * if element in matrix is non-zero)

(b) [1.2] Determine the minimum degree ordering and sketch the matrix after reordering. Also motivate your choices.

(c) [0.8] Sketch the L factor of Gaussian elimination without pivoting of the matrix in the previous part. Motivate your answer.